

Math Virtual Learning

AP Statistics

April 15th, 2020

Lesson: April 15th, 2020

Objective:

- 1. Students will understand the connection between Significance Tests & Confidence Intervals**
- 2. Students will understand and solve problems with computer-generated output**

Review #1: [Formula Sheet with Table A](#)

Reject or Fail to Reject the H_0 ?

$$\alpha = .05$$

$$H_0: p = .25$$

$$H_a: p \neq .25$$

$$z = 1.40$$

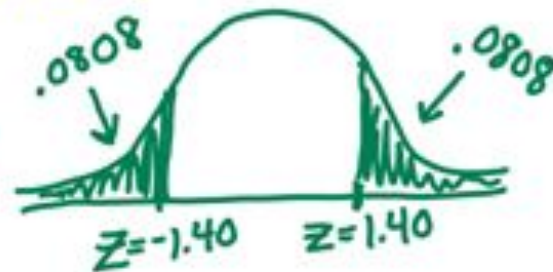
$$\text{p-value} = \underline{\hspace{2cm}}$$

Review #1 Answer:

$$H_0: p = .25$$

$$\alpha = .05$$

$$H_a: p \neq .25 \rightarrow \text{two-tailed test}$$



$$z = 1.40$$

$$\text{p-value} = \underline{2(.0808)} = .1616$$

Since the p-value is greater than $\alpha = .05$ we
Fail to Reject the H_0 and cannot conclude the H_a .

Review #2: [Formula Sheet with Table A](#)

Reject or Fail to Reject the H_0 ?

$$H_0: p = .60$$

$$\alpha = .05$$

$$H_a: p < .60$$

$$z = -1.93$$

$$\text{p-value} = \underline{\hspace{2cm}}$$

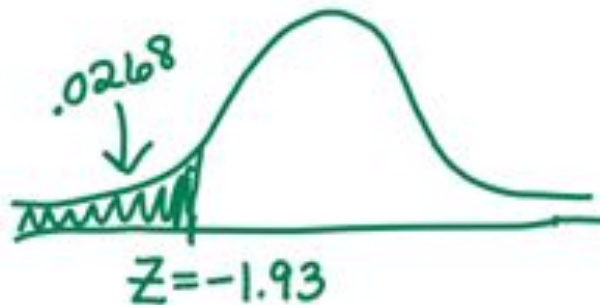
Review #2 Answer:

$$H_0: p = .60$$

$$\alpha = .05$$

$$H_a: p < .60 \rightarrow \text{one-tailed test}$$

less than



$$z = -1.93$$

$$\text{p-value} = \underline{\underline{.0268}}$$

Since the p-value is less than $\alpha = .05$ then we Reject the H_0 and conclude H_a .

The Connection Between Significance Levels & Confidence Intervals

Nonsmokers $\hat{p} = \frac{90}{150} = .6$

$p = .50$

According to the Centers for Disease Control and Prevention (CDC) Web site, 50% of high school students have never smoked a cigarette. Tim wonders whether this national result holds true in his large, urban high school.

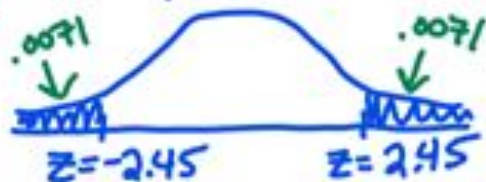
For his AP Statistics class project, Tim surveys an SRS of 150 students from his school. He gets responses from all 150 students, and 90 say that they have never smoked a cigarette. What should Tim conclude? Give appropriate evidence to support your answer. (Use the 4-step process). \rightarrow only some parts shown here

$$H_0: p = .50$$

$$H_a: p \neq .50 \rightarrow \text{two-tailed test}$$

$$\alpha = .05$$

$$Z = \frac{.6 - .50}{.0408} = 2.45$$



$$p\text{-value} = 2(.0071) = .0142$$

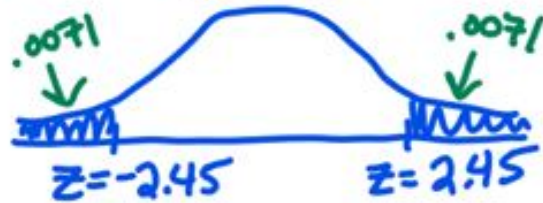
What can you conclude?

$$H_0: p = .50$$

$$H_a: p \neq .50 \rightarrow \text{two-tailed test}$$

$$\alpha = .05$$

$$Z = \frac{.6 - .50}{.0408} = 2.45$$



$$p\text{-value} = 2(.0071) = .0142$$

What can you conclude?

Conclude: Since the p-value, .0142, is less than $\alpha = .05$, we Reject the H_0 and can conclude the H_a . Thus, there is statistically significant evidence to conclude that the % of nonsmokers is different than .50.

We just completed a 5% significance test (since alpha was .05) for the true proportion of high school students who have never smoked a cigarette. Now we will construct a 95% confidence interval to estimate the same parameter. Will our interval support the result of our significance test?

Note: 5% + 95% = 100%. If we had done a 10% significance test we would construct a 90% confidence interval.

$$CI = \hat{p} \pm z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

remember to use \hat{p} instead of p
in a confidence interval

$$= .6 \pm 1.96 \cdot \sqrt{\frac{.6(.4)}{150}}$$

$$= .6 \pm .0784$$

$CI = (.5216, .6784)$ We are 95% confident that this interval captures the true proportion.

Remember the $H_0: p = .50$? Well .50 is NOT in the interval. So, the CI supports our conclusion to Reject the H_0 .

Example:

Reject or Fail to Reject the H_0 ?

$H_0: p = .32$ **vs.** $H_a: p \neq .32$ with $\alpha = .05$

The 95% confidence interval is (.37, .63)

Example:

$H_0: p = .32$ vs. $H_a: p \neq .32$ with $\alpha = .05$

The 95% confidence interval is (.37, .63)

Reject the H_0 , conclude the H_a

← .32 is
NOT in
the
interval

Practice #1:

Reject or Fail to Reject the H_0 ?

$H_0: p = .60$ **vs.** $H_a: p < .60$ with $\alpha = .05$

The 95% confidence interval is (.56, .84)

Practice #1 Answer

$H_0: p = .60$ vs. $H_a: p < .60$ with $\alpha = .05$

The 95% confidence interval is (.56, .84)

Fail to Reject the H_0 , cannot conclude the H_a .

\nearrow .60 is in the interval
so it is a plausible
value for p

Practice #2

Reject or Fail to Reject the H_0 ?

$H_0: p = .10$ **vs.** $H_a: p > .10$ with $\alpha = .01$

The 99% confidence interval is (.11, .33)

Practice #2 Answer

$H_0: p = .10$ vs. $H_a: p > .10$ with $\alpha = .01$

The 99% confidence interval is (.11, .33)

Reject the H_0 , conclude H_a .

↑ .10 not in
interval

Understanding Computer-Generated Output

Session							
Test and CI for One Proportion							
Test of $p = 0.5$ vs $p \text{ not} = 0.5$ ($p \neq .50$)							
H_0 vs H_a							
Sample	X	N	Sample p	95% CI	Z-Value	P-Value	
1	90	150	0.600000	(0.521601, 0.678399)	2.45	0.014	

test statistic

of successes

sample size

\hat{p}

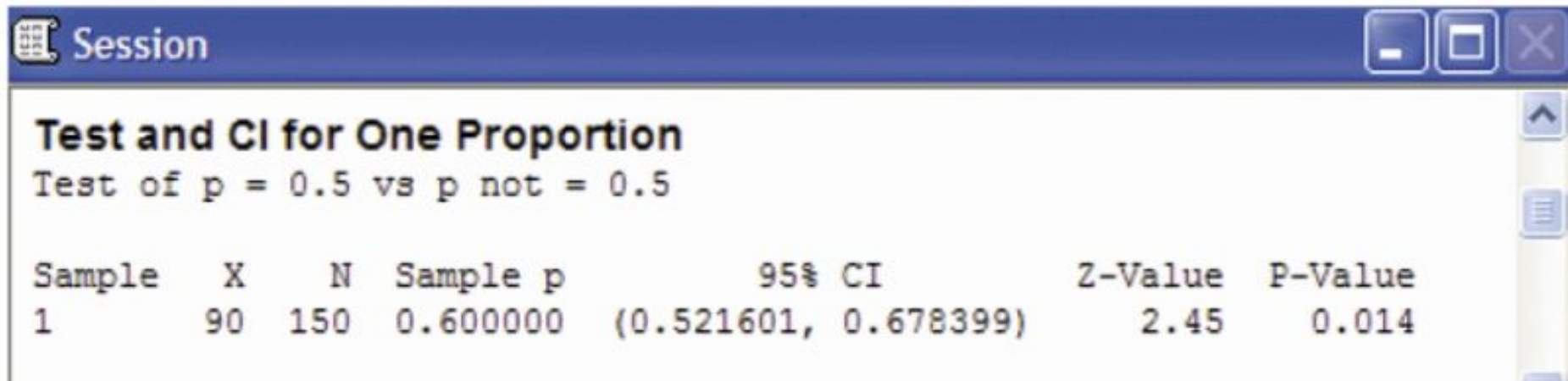
plausible values of p

★ $p=0.5$ is not in the interval

Both the CI & the p-value agree that $p \neq .50$

★ this is less than $\alpha = .05$ so we Reject the H_0 & conclude $p \neq .50$

Understanding Computer-Generated Output



The image shows a screenshot of a Minitab 'Session' window. The title bar is blue with the word 'Session' and standard window controls. The main area has a black background with white text. The title 'Test and CI for One Proportion' is in bold. Below it, the hypothesis test is stated as 'Test of p = 0.5 vs p not = 0.5'. A table follows with columns for Sample, X, N, Sample p, 95% CI, Z-Value, and P-Value. The data row shows a sample proportion of 0.600000 with a 95% confidence interval of (0.521601, 0.678399), a Z-value of 2.45, and a P-value of 0.014.

Sample	X	N	Sample p	95% CI	Z-Value	P-Value
1	90	150	0.600000	(0.521601, 0.678399)	2.45	0.014

The confidence interval in this example is much more informative than the significance test we performed earlier. The interval gives the values of p that are consistent with the sample data. We would not be surprised if the true proportion of students at Taeyeon's school who would say they have never smoked cigarettes was as low as 52.2% or as high as 67.8%. However, we would be surprised if the true proportion was 0.50 because this value is not contained in the confidence interval. **Figure 9.11** gives computer output from Minitab software that includes both the results of the significance test and the confidence interval.



CHECK YOUR UNDERSTANDING

The figure below shows Minitab output from a significance test and confidence interval for the restaurant worker data in the previous Check Your Understanding ([page 558](#)). Explain how the confidence interval is consistent with, but gives more information than, the test.

Test and Confidence Interval for One Proportion						
Test of $p = 0.75$ vs $p \text{ not } = 0.75$						
Sample	X	N	Sample p	95.0 % CI	Z-Value	P-Value
1	68	100	0.680000	(0.588572, 0.771428)	-1.62	0.106

Correct Answer

In the previous Check Your Understanding, we failed to reject the null hypothesis that the proportion of restaurant employees at this chain who say that work stress has a negative impact on their personal lives is the same as the national proportion of 0.75. The confidence interval given in the output includes 0.75, which means that 0.75 is a plausible value for the population proportion that we are seeking. So both the hypothesis test (which didn't rule out 0.75 as the proportion) and the confidence interval (which gave 0.75 as a plausible value) give the same conclusion. The confidence interval, however, gives more information in that it gives a whole range of plausible values, whereas the hypothesis test concentrates only on the one value as a possibility for the population proportion (0.75 here).

More Practice

p. 564-565 #55, 57, 59, 60